with

$$Q_1 = J_{k+n}\left(\frac{x'_{nm}d}{a_2}\right)C_1(k+n) + (-1)^n J_{k-n}\left(\frac{x'_{nm}d}{a_2}\right)C_2(k-n) \quad (21)$$

$$Q_2 = -J_{k+n}\left(\frac{x_{nm}d}{a_2}\right)C_2(k+n) + (-1)^n J_{k-n}\left(\frac{x_{nm}d}{a_2}\right)C_1(k-n) \quad (22)$$

$$Q_3 = J_{k+n}\left(\frac{x_{nm}d}{a_2}\right)C_3(k+n) + (-1)^n J_{k-n}\left(\frac{x_{nm}d}{a_2}\right)C_4(k-n) \quad (23)$$

$$C_1(k+n) = \begin{bmatrix} -(\epsilon_k - 1)\cos((k+n)\theta) & \sin((k+n)\theta) \\ (\epsilon_k - 1)\sin((k+n)\theta) & \cos((k+n)\theta) \end{bmatrix} \quad (24)$$

$$C_2(k-n) = \begin{bmatrix} (\epsilon_k - 1)\cos((k-n)\theta) & -\sin((k-n)\theta) \\ (\epsilon_k - 1)\sin((k-n)\theta) & \cos((k-n)\theta) \end{bmatrix} \quad (25)$$

$$C_3(k+n) = \begin{bmatrix} \cos((k+n)\theta) & (\epsilon_k - 1)\sin((k+n)\theta) \\ \sin((k+n)\theta) & -(\epsilon_k - 1)\cos((k+n)\theta) \end{bmatrix} \quad (26)$$

$$C_4(k-n) = \begin{bmatrix} \cos((k-n)\theta) & (\epsilon_k - 1)\sin((k-n)\theta) \\ -\sin((k-n)\theta) & (\epsilon_k - 1)\cos((k-n)\theta) \end{bmatrix} \quad (27)$$

and $\epsilon_k = 1$ for $k = 0$, and 2 for $k > 0$.

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Modellization of Losses in TE_{011}-Mode Waveguide Bandpass Filters

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***Abstract*—A mode-matching technique for the analysis of TE_{011} mode waveguide cylindrical bandpass filters including losses is presented. The modes of a lossy radial waveguide are derived and the generalized scattering matrix of the lossy cavity coupled by two rectangular apertures is computed enforcing an impedance boundary condition on the cavity sidewall. Cavity sidewall losses as well as top and bottom wall losses are therefore taken accurately into account. Numerical and experimental results are given for a four cavity filter in K_a band.**

I. INTRODUCTION

Cylindrical cavities resonating in TE_{011}-mode are very attractive for the realization of low-loss narrow-band filters. Fig. 1 shows the structure of a filter section: cylindrical cavities are coupled together and to the external waveguide by means of short rectangular coupling irises operating below cutoff. The two apertures on the cavity sidewall form an angle 2ϑ.

In [1], [2] the authors presented a mode-matching technique to analyse accurately this kind of filters. That procedure allowed to take accurately into account the effects of the thick coupling apertures, the irises angular offset 2ϑ, the spurious responses and the higher mode interaction between adjacent resonators, overcoming the limitations of available approximate models [3], [4]. After that, by optimization procedures it is possible to design filters having the desired frequency response without resorting to empirical adjustments.

In the present paper, it is explained how to modify this mode matching technique to take into account also ohmic losses. Top, bottom, and cavity sidewall are assumed to have finite conductivity while coupling irises, which are very short and operate below the cutoff, are assumed lossless. Moreover, since in the passband the field configuration inside the cavity is very similar to the TE_{011} mode only, losses due to currents flowing in the x-direction, which are due only to spurious modes, are neglected.

For sake of simplicity in this paper the analysis is limited to cavities with two identical apertures symmetrically placed with respect to the height of the cavity. The general case of cavities with two different apertures can be derived with minor modifications of the algorithm.

Sections II and III reports the formal solution of the field problem and Section IV some numerical and experimental results.

II. STATEMENT OF THE PROBLEM

The analysis of TE_{011} bandpass filters is conveniently carried out by splitting the whole structure in simpler building blocks, as shown in Fig. 1. Two discontinuities must be analyzed: the symmetrical double-step formed by the junction between the rectangular external waveguide and the first (last) rectangular coupling iris and the discontinuity at the junction between the irises and the cavity itself. Each discontinuity is considered separately and its generalized scattering matrix is computed. The overall scattering matrix of the total filter is hence obtained by a suitable direct combination of all single scattering matrices [5]. The analysis of the double-step discontinuity in rectangular waveguide is a well known problem, efficiently solved

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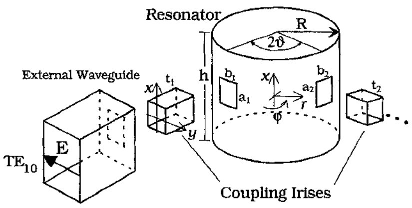

Fig. 1. Schematic view of the TE_{011} bandpass filter.

by several authors [6], [7] using a TE^x field expansion and is not treated in this context. The transverse field components in the rectangular aperture are

$$E_y^{\Box} = j\omega\mu \sum_{u=1}^{\infty} \left(a_u^+ + a_u^-\right)\phi_u^{\Box}(x,y) \tag{1}$$

$$H_x^{\Box} = \sum_{u=1}^{\infty} \left(a_u^+ - a_u^-\right)\left(k^2 - k_{xp}^2\right)\phi_u^{\Box}(x,y)/\gamma_u \tag{2}$$

being $\gamma_u = \sqrt{k_{xp}^2 + k_{yq}^2 - k^2}$ the propagation constant, $k^2 = \omega^2\mu\varepsilon$, $k_{xp} = (2p+1)\pi/a$ and $k_{yq} = q\pi/b$ with $p, q = 0, 1, 2, \ldots \infty$. Finally a_u^+ and a_u^- are the incident and reflected modal coefficients and $\phi_u^{\Box}(x,y)$ is the rectangular mode function

$$\phi_u^{\Box}(x,y) = \sqrt{\frac{2\epsilon_q}{ab}} \cos k_{xp}x \cos k_{yq}\left(\frac{b}{2} + y\right). \tag{3}$$

ϵ_q is the Neumann factor and u is some combination of p and q.

Also in the cavity only TE^x modes (simply TE in the following) are considered, because of the orientation of the external waveguide (see Fig. 1). Although in practice TM modes can be weakly excited, in general their effects are negligible [4]. Considering the cavity as a radial waveguide of height h with lossy plates, the modal fields components of interest are

$$E_\varphi^{\circ} = j\omega\mu \sum_{v=1}^{\infty} c_v \frac{J_i'(\kappa_{cn}r)}{\kappa_{cn}} \phi^{\circ}_{v^e_m}(\varphi, x) \tag{4}$$

$$H_x^{\circ} = \sum_{v=1}^{\infty} c_v J_i(\kappa_{cn}r)\phi^{\circ}_{v^e_m}(\varphi, x) \tag{5}$$

$$H_r^{\circ} = \sum_{v=1}^{\infty} c_v \frac{J_i'(\kappa_{cn}r)}{\kappa_{cn}} \frac{\partial\phi^{\circ}_{v^e_m}(\varphi, x)}{\partial x} \tag{6}$$

where c_v is the modal amplitude coefficient and $\phi^{\circ}_{v^e_m}(\varphi, x)$ is the modal eigenfunction

$$\phi^{\circ}_{v^e_m} = \sqrt{\frac{\epsilon_i}{hr\pi}} \cos(\overline{\kappa}_{xn}x)^{\cos}_{\sin}(i\varphi) \tag{7}$$

solution of the homogeneous Helmholtz equation satisfying the boundary condition

$$\mathbf{E}_\varphi^{\circ} = \mathcal{Z}_s \hat{\mathbf{x}} \times \mathbf{H}_r^{\circ} \quad \text{at} \quad x = \pm h/2. \tag{8}$$

Condition (8) is rigorous only for TE_{0nm}-modes. For a good conductor the surface impedance is $\mathcal{Z}_s = (1+j)\sqrt{\omega\mu_0/2\sigma}$ with σ being the conductivity of the wall metal. In (4)–(7) $i, n = 0, 1, 2 \cdots \infty$ and $\kappa_{cn} = \sqrt{k^2 - \overline{\kappa}_{xn}^2}$. The eigenvalue $\overline{\kappa}_{xn}$ is calculated by means of the boundary condition (8) and, by a limiting process, is found as

$$\overline{\kappa}_{xn} \simeq \kappa_{xn}\left[1 - (1-j)\frac{\delta}{h}\right] \tag{9}$$

where δ is the skin depth and $\kappa_{xn} = (2n+1)\pi/h$ is the eigenvalue of the perfectly conducting case.

The termination of such a lossy waveguide with an impedance surface with two aperture at a distance from the center of the cavity equal to its radius R is now enforced by the mode matching technique and the generalized scattering matrix of the resonator is obtained.

III. The Scattering Matrix of the Lossy Cavity

By taking advantage of the symmetry, only half cavity, obtained by inserting either an electric or a magnetic wall at the symmetry plane, can be analyzed. The two generalized one port scattering matrices of half the cavity are derivable by suitable projection of the E-field and H-field continuity equations over the aperture.

If b is small compared with R, the derivation of the two equations is straightforward and the continuity equations write

$$E_\varphi^{\circ} = \begin{cases} E_y^{\Box} & \text{on aperture } S_A \\ \mathcal{Z}_s H_x^{\circ} & \text{on cavity sidewall } S_W \end{cases} \tag{10}$$

$$H_x^{\circ} = H_x^{\Box} \qquad \text{on aperture } S_A. \tag{11}$$

Equation (10) is projected by using the cavity eigenfunctions $\phi^{\circ}_{v^e_m}(\varphi, x)$ over the cavity sidewall S_W and on the aperture S_A. The continuity equation of the magnetic field (11) is projected using the eigenfunctions $\phi_u^{\Box}(x,y)$ of the rectangular coupling iris on the aperture surface S_A. This leads to the matrix equations

$$\begin{cases} \frac{1}{2}\mathbf{C} = \mathbf{H}_{\substack{e\\m}}(\mathbf{A}^+ + \mathbf{A}^-) + \mathcal{Z}_s \mathbf{W}_{\substack{e\\m}} \mathbf{Y}^{\circ}\mathbf{C} \\ \mathbf{H}^T_{\substack{e\\m}} \mathbf{Y}^{\circ}\mathbf{C} = \mathbf{Y}^{\Box}(\mathbf{A}^+ - \mathbf{A}^-) \end{cases} \tag{12}$$

where $\mathbf{A}^+ = [a_u^+]$, $\mathbf{A}^- = [a_u^-]$ and $\mathbf{C} = [c_v]$ are the incident, reflected and the cavity modal amplitude vectors coefficients, $\mathbf{H}_{\substack{e\\m}}$ and $\mathbf{W}_{\substack{e\\m}}$ are the coupling matrices and $\mathbf{Y}^{\Box}$ and $\mathbf{Y}^{\circ}$ are two diagonal square matrices of normalizing coefficient. Superscript 'T' denotes transpose. The elements of the coupling matrices are defined as

$$H_{\substack{e\\m}}(v,u) = \int_{S_A} \phi^{\circ}_{v^e_m}(\varphi, x)\phi_u^{\Box}(x,y)ds, \tag{13}$$

$$W_{\substack{e\\m}}(v,u) = \int_{S_W} \phi^{\circ}_{v^e_m}(\varphi, x)\phi^{\circ}_{u^e_m} ds, \tag{14}$$

$$Y^{\Box}(u,u) = \frac{k^2 - k_{xp}^2}{\gamma_u} \tag{15}$$

and

$$Y^{\circ}(v,v) = \frac{\kappa_{cn}J_i(\kappa_{cn}R)}{J_i'(\kappa_{cn}R)} = \frac{\kappa_{cn}^2 R}{i - \kappa_{cn}R\frac{J_{i+1}(\kappa_{cn}R)}{J_i(\kappa_{cn}R)}}. \tag{16}$$

Integration (13) can be carried out by replacing $\overline{\kappa}_{xn}$ with κ_{xn} without loosing in accuracy and integration (14) is avoided by observing that [8]

$$\mathbf{W}_{\substack{e\\m}} = \frac{1}{2}\mathbf{I} - \mathbf{H}_{\substack{e\\m}}\mathbf{H}^T_{\substack{e\\m}}. \tag{17}$$

By solving the two matrix (12) as usual, the two scattering one port matrices $\mathbf{\Gamma_e}$ and $\mathbf{\Gamma_m}$ are obtained

$$\mathbf{\Gamma}_{\substack{e\\m}} = \left(\mathbf{Y}^{\Box} + 2\mathbf{H}^T_{\substack{e\\m}}\mathbf{Y}^{\circ}\mathbf{Z}\mathbf{H}_{\substack{e\\m}}\right)^{-1}\left(\mathbf{Y}^{\Box} - 2\mathbf{H}^T_{\substack{e\\m}}\mathbf{Y}^{\circ}\mathbf{Z}\mathbf{H}_{\substack{e\\m}}\right) \tag{18}$$

where

$$\mathbf{Z} = (\mathbf{I} - 2\mathcal{Z}_s\mathbf{W}_{\substack{e\\m}}\mathbf{Y}^{\circ})^{-1} \simeq \mathbf{I} + 2\mathcal{Z}_s\mathbf{W}_{\substack{e\\m}}\mathbf{Y}^{\circ} \tag{19}$$

and hence the scattering matrix of the symmetrical resonator is

$$\mathbf{S}_{11} = \frac{\mathbf{\Gamma}_m + \mathbf{\Gamma}_e}{2}, \quad \mathbf{S}_{21} = \frac{\mathbf{\Gamma}_m - \mathbf{\Gamma}_e}{2}. \tag{20}$$

IV. Numerical and Experimental Results

This technique has been used to investigate the influence of the finite conductivity of the cavity walls on the frequency behavior near the resonance of the TE_{011} mode. Fig. 2 shows the computed

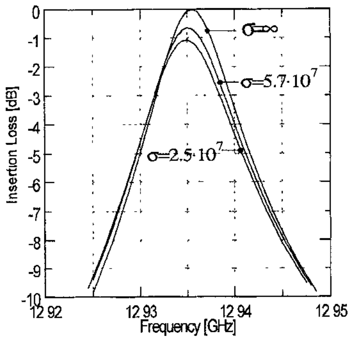

Fig. 2. Computed insertion loss of a single cavity filter.

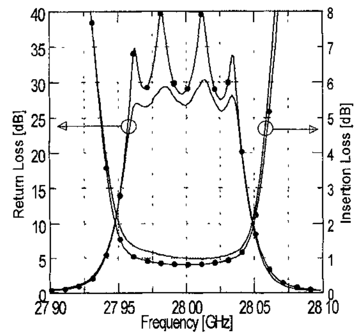

Fig. 3. Comparison between measured (—) and calculated (—•—•—) insertion and return loss of a four cavity filter.

TABLE I
THEORETICAL AND CALCULATED UNLOADED Q_J

σ	Q	Q_J	Q_J [10]
5.7×10^7	2940	25100	23649
2.5×10^7	2800	17590	15662

transmission coefficient through a single cavity filter with dimensions $h = 20.5$ mm, $D = 34$ mm, $2\vartheta = 180°$, $a = 8$ mm, $b = 4$ mm and $t = 3$ mm, feeds with an R120 rectangular waveguide.

The frequency responses of the cavity with $\sigma = 5.7 \times 10^7$ S/m and $\sigma = 2.5 \times 10^7$ S/m are shown. The former is the theoretical conductivity of copper and the latter corresponds to a value measured in the range $16 \div 28$ GHz [9]. The lossless filter response is reported too showing a resonant frequency $f_0 = 12.936$ GHz and a loaded $Q = 3330$. The loaded Q has been calculated as $Q = 2f_0/B$, where B is the 3 dB bandwith, and the unloaded Q_J for a completely closed lossy cylindrical cavity derived. Table I reports the loaded Q of the lossy cavity and a comparison between the unloaded Q_J evaluated by this technique and the theoretical Q_J of the TE_{011} mode of the same cylindrical cavity. The agreement is satisfactory considering also the difficulties of an accurate evaluation of the bandwidth B.

A four cavities filter is now considered. The center frequency is 28 GHz and the bandwidth is 80 MHz. The filter has been designed with the approximated methods available in literature [3], [4] and then optimized resorting to the mode-matching technique. Cavities dimensions in mm are $D = 15, h_1 = 9.997, h_2 = 9.954, 2\vartheta_1 = 135°$ and $2\vartheta_2 = 90°$ and coupling irises dimensions are $a = 4.6, b = 2, t_1 = 2.539, t_2 = 7.398$ and $t_3 = 8.628$. R320 external waveguide has been used to feeds the filter. A comparison between calculated and measured insertion loss is reported in Fig. 3. A good agreement is observed also on the insertion loss in the passband. In the simulation the *measured* value $\sigma = 2.5 \times 10^7$ S/m has been used.

V. CONCLUSION

This paper extend the mode matching technique used for the analysis of cylindrical cavity TE_{011} mode bandpass waveguide filters to include the effects of finite conductivity of the resonators. Only minor changes to the computer code of the lossless case are necessary. Cavity sidewall losses as well as end walls losses are accurately taken into account. Experimental results on a four cavity filter fully validate this approach.

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